Provably Secure S-Box Implementation Based on Fourier Transform

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Overview

- Differential Power Analysis on block ciphers
- Notion of DPA-resistance
- A new method to protect S-Box
- Application to AES
- Conclusion

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    - Duplication method
    - S-Box secure calculation

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- $Adv(\mathcal{M}) = 0 \Longleftrightarrow$ all the variables at the unit level of $\mathcal{M}$ are independent from the sensitive input.

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- As $\widehat{\widehat{F}} = 2^n F$, we have:

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DPA-resistant S-Box Implementation

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- Mask correction performed *on-the-fly*.

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- New formula:

$$\boxed{(-1)^{(\widetilde{X}\oplus R_2)\cdot R_1} F(X) + R_3 = \left\lfloor \frac{1}{2^n} \left(R' + \sum_{A \in \mathbb{F}_2^n} \widehat{F}(A)(-1)^{A\cdot\widetilde{X} \oplus R_1\cdot(\widetilde{X}\oplus A \oplus R_2)} \right) \right\rfloor}$$

where $R_2, R_3, R_4 \in \mathbb{F}_2^n$ and $R' = 2^n R_3 + R_4$.

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- Procedure based on Goubin's method:

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$$(X \oplus R_1, R_1, \widehat{F}) \mapsto (F(X) \oplus R_3, R_3)$$

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- Efficiency: exponential in the dimension of the S-Box

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$\Rightarrow$ Tower Field Methods

- Down to $\mathbb{F}_{2^4}$ and apply our method to protect inversion

AES: implementation results

Comparison of several methods to protect AES against DPA:

Method	Timings (ms)	RAM (bytes)	ROM (bytes)
Straightforward implementation	5	32	1150
This paper	32	39	3100
Oswald *et al.* (FSE'05)	26	42	3400
Trichina *et al.* (WISA'04)	21	291	3050

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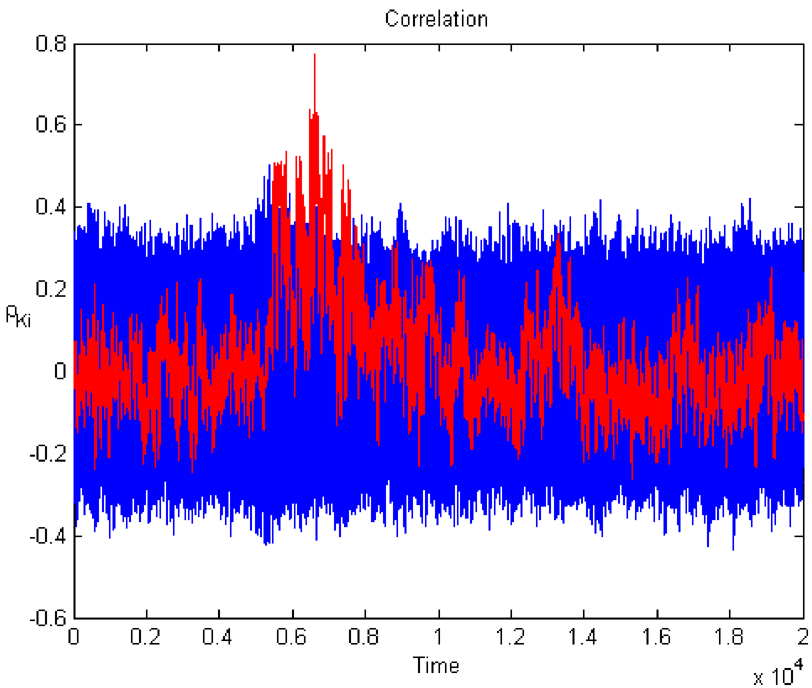
CPA on straightforward method

using 100 random plaintexts



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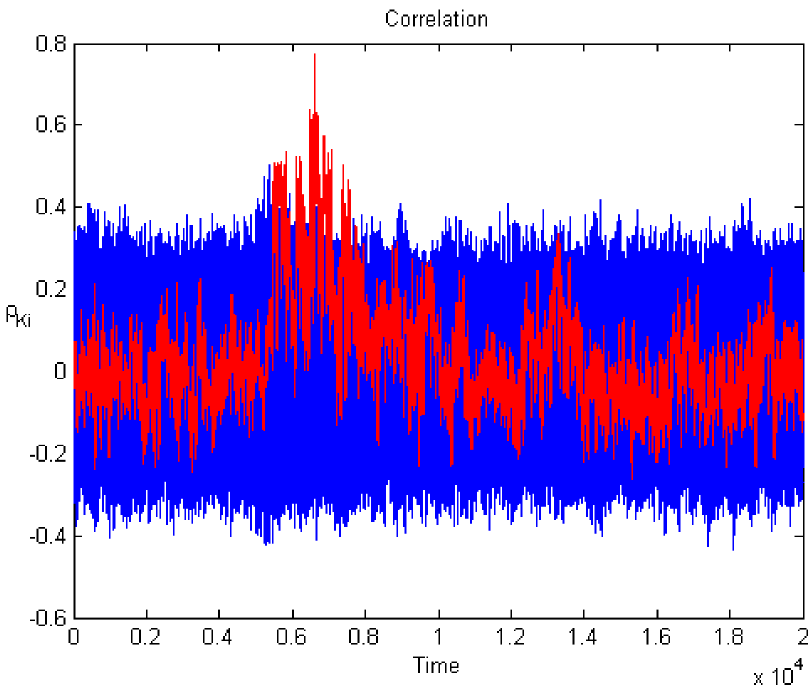
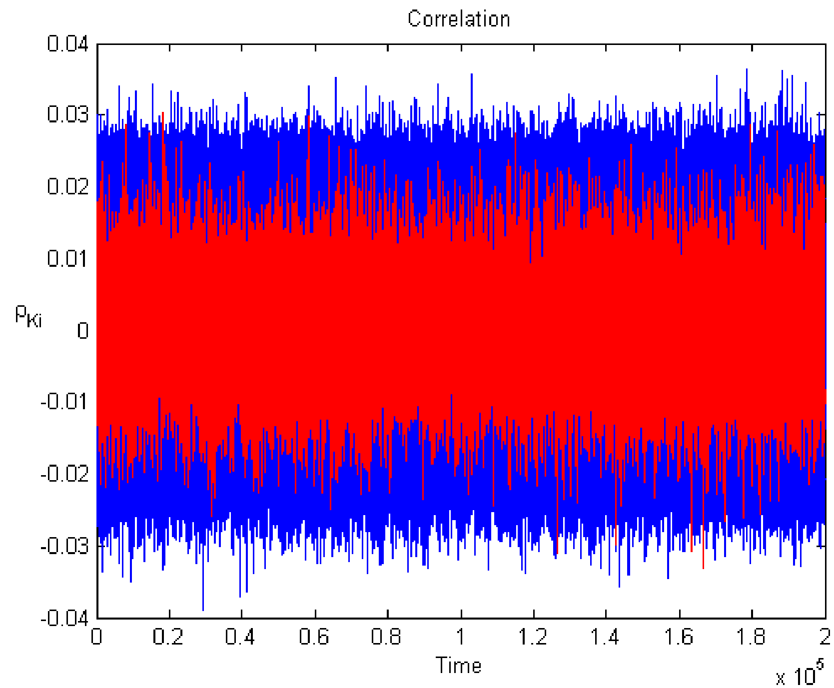


CPA on our method

using $20\,000$ random plaintexts



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- Perspectives:
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  - Find other transformations than the Fourier Transform

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